THE CONFIGURATION SPACE OF A PARALLEL MECHANISM

1. Introduction

A *(single) branch* is a concatenation of n rigid rods joined by rotational joints where one of the end joints (*base joint*) is fixed, the other end is *free*. A spider mechanism is thus simply k branches connected at their free joints. Parallel robots are of great interest due to their accuracy and stiffness properties (for example, take the planar spider like robot presented in [SRS]), thus applicable to medicine and research (see [GS]).

A mechanism *configuration* is a set of independent parameters uniquely specifying the position of each of its mechanical component. A mechanism *configuration space* is thus simply the set of all configurations. For example, a simple n-linked branch mechanism has the n-torus $\mathbf{T}^n$ as its configuration space. The configuration space is very important in areas such as robotics, where motion planning problems are essential, i.e., moving from an initial point to an end point, taking into account constraints (see, e.g., [GA]).

In the last few years much work has been done on finding the configuration space of some simple spider mechanisms, such as a *single loop* mechanism, where each vertex corresponds to a rotational mechanical joint (see [HK,HR]). There is a strong correspondence between the topological properties of any mechanism configuration space (the structure of its cohomology algebra) and the character of motion planning instabilities (see [F]). In this note we deal with spider mechanisms in plane which have an arbitrary number of branches with a common point. A branch configuration $\mathcal{V} = (v_1, .., v_n)$ is *aligned* (with direction w) if each vector $v_1, \ldots, v_n$ is a scalar multiple of w. A configuration $\mathcal{V} = (V^{(1)}, .., V^{(k)})$ of $(\mathcal{L}, \mathcal{X})$ is called a *k-node* if it has k aligned branch configurations with directions $w_1, \ldots, w_k$ which are linearly dependent; otherwise it is called *generic* In §3 we show that a configuration space of a generic mechanism is a smooth orientable surface, – in which cases the configuration space contains pinch points. Thus the Euler characteristic for each connected component completely determines the topology of the configuration space. As an example, we present a computer aided complete list of all *cells* in the parameter space and their corresponding configuration spaces for a 3-branched mechanism where the fixed ends are all aligned.

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