CONNECTIVITY IN OPEN AND CLOSED LOOP ROBOTIC MECHANISMS

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ABSTRACT

This paper deals with determining linkages which have a connectivity of six between at least two of the links. Such linkages are useful in the design of mechanical arms. Using the basic concepts embodied in the Grübler mobility formula, we survey the possibilities for structural arrangements which yield six-degree-of-freedom manipulators. Then we develop the concept of link connectivity, and show how to produce modified graphs which facilitate computing link connectivity. In the modified graphs, link connectivity is shown to be equivalent to the weighted distance between graph vertices.

1. INTRODUCTION

1a. Number Synthesis of Mechanisms

Number synthesis answers the question: what are the possible number of ways of combining links and kinematic joints to obtain a given mechanism mobility? The basic tool is the formula:

(1) Where M is the mechanism's mobility (i.e., the total degrees of freedom which need to be controlled in the mechanism for every link to be in a specific position), n is the number of rigid links including the base or frame (or an equivalent number of rigid links if the mechanism has non-rigid members), f_i is the freedom in the ith joint, and p is the total number of joints [Angeles, 1988; Franke, 1943]. This formula, usually attributed to Grübler and Kutzbach, allows the basic calculation for a mechanism's mobility, which we often call its degrees of freedom. In general we use the equality sign to determine M, the greater–than is used only when the mechanism has special proportions (see, for example, Hunt [1978; 1983]).

In robotics this formula has been used to determine possible structures for manipulators, hands, and walking machines [Sakamoto et. al., 1993]. However, we need to be cautious, since in devices like manipulators we are not generally concerned with the mechanism's mobility but rather the number of degrees–of–freedom between two specific links — usually, the freedom between the ground link and the end–effector link. The degree–of–freedom between two links is called their *connectivity* [Phillips, 1984]. Each link in a mechanism has a connectivity with respect to every other link. If two links are jointed together the connectivity equals the f_i for that joint. For non adjacent links the connectivity is upper bounded by the mechanism's mobility.

1b. Series open–chains

In its simplest form a mechanism can be composed of an open series–chain with each member simply jointed to a single previous–link neighbor and a subsequent–link neighbor. Since the first link has no previous link and the last link has no subsequent link, such series chains clearly have one less joint than the number of links, i.e., p = n–1. Hence for such mechanisms $M = \sum_{i=1}^{p} f_i$. This is the basis for all the common manipulator structures with series chains and it verifies the intuitive result that if freedoms are simply strung out in series the resulting system has the total freedom of the individual joints. Further, if all the joint freedoms are independent, the connectivity between the first and last link in the chain is equal to the mobility. (For discussion of mechanisms having special proportions and hence geometry-depended mobility see Hunt [1978]).

1c. Closed loops chains

For a single closed loop, n=p and therefore (1) yields:

$M = -6 + \sum_{i=1}^{p} f_i$. This tells us that, if we use independent one–degree–of–freedom joints, we need seven joints (and links) to have a mobility of one in a single–loop closed chain. Beyond seven joints, the mobility increases by one for every additional joint.

If we introduce one or more closed chains within the mechanism, the mobility formula must be used in its most general form, (1). By decomposing a mechanism into a single simple closed loop, for which n=p, and a group of in–parallel series chains, (1) can be modified to take into account the relation that n=p–1 in each in–parallel series chain added to the initial closed loop. Therefore, (1) becomes $M = \sum_{i=1}^{p} f_i - 6l$, where l is the total number of independent loops.

For example, the in–parallel chain (incorrectly) attributed to Stewart, and referred to as the general Stewart platform, has six linear actuators (also called telescoping “links”), each with a ball–and–socket (spherical) joint connection to both the ground and the coupler (or platform) link [Merlet, 1990]. Since each linear actuator is actually composed of two links connected by a one–degree–of–freedom sliding (or prismatic) joint, we have n=14, p=18, and $\sum_{i=1}^{p} f_i$=42. With these values formula (1) gives M=12. Alternatively, since there are five independent loops, l=5, and the loop formula also gives M=12.

Continuing with this example, since six of these freedoms are simply the possibility of rotating each linear actuator about the line connecting the centers of its two spherical joints, we are left with six freedoms which can be used to control the linear actuators. Once these are specified there is no freedom left for the coupler to move. Hence we speak of this as a six–degree–of–freedom mechanism, even though its mobility is actually twelve. More accurately, we should say that the mechanism’s freedom (or mobility) is twelve, but the connectivity between coupler and ground is six. It should be mentioned that the fact that each linear actuator can rotate about the line connecting its two spherical joints is due to the special geometry of the spherical joints. Clearly, mobility equations, (1), and connectivity matrices (Section 7) that do not take into consideration special geometry can give erroneous results.

2. SIX-DEGREES-OF-FREEDOM MECHANISMS

Clearly if we use the general formula we can “invent” thousands of different manipulator configurations [Hunt and Primrose, 1993]. If we want six–degree–of–freedom linkages composed of only one–degree–of–freedom joints, formula (1) gives:

$$12 - 6n + 5p = 0. \qquad (2)$$

From which we get the following table:

n	p
7	6
12	12
17	18
22	24
27	30
32	36
37	42
.	.
.	.

This table goes on indefinitely, requiring ever greater numbers of links to form the mechanism. However, it should be pointed out that since we are only studying six–degree–of–freedom mechanisms, we will require only six actuators regardless of how many links are in the mechanism. Each entry in the table can be realized by different combinations of link and joint types. If we restrict ourselves to, say, one–degree–of–freedom joints of the revolute type, the first row in the table (n=7 and p=6), represents the ubiquitous open–loop, seven–link, revolute series–chain configuration, Figure 1a.

In Figure 1a, and in Figures 1b-9, we use the thicker lines to represent links and thinner lines for joints; shaded polygons indicate links with more than two joints.

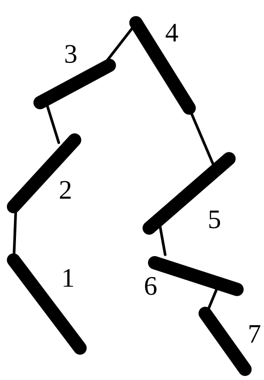


Fig. 1a

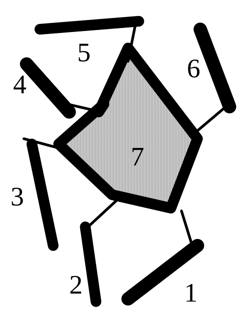


Fig. 1b

Conventionally, link 1 is the base of the manipulator and link 7 is part of the end–effector. Structurally, any link can be the “fixed” base, so if we fix an intermediate link the chain still has six–degrees–of–freedom, but now no one link has a connectivity of six relative to the base. In general, if we are interested in six–degree–of–freedom manipulators we can only use mechanisms where there are (at least) two links with relative connectivity of six; we can then make one of these the base and the other the end-effector.

If this condition is not satisfied, then even if the mechanism mobility is six (or even more), some of the freedoms are “useless” from the point of view of manipulation, and the mechanism does not satisfy the necessary requirements. For example, the first row in the table also represents the configuration shown in Figure 1b, which would be rather useless as a conventional manipulator since the connectivity between links is either one or two. Of course such a mechanism might have other uses in robotics, perhaps in walking, gripping or conveyor type operations. In Section 7 we give a simple algorithm for identifying a link's connectivities.

Working our way down the table, and assuming only revolute joints, we can immediately identify the following useful six–degree–of–freedom mechanisms:

The second row is the twelve–link closed–loop series chain, Figure 2.

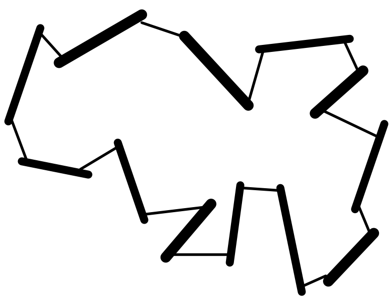


Figure 2

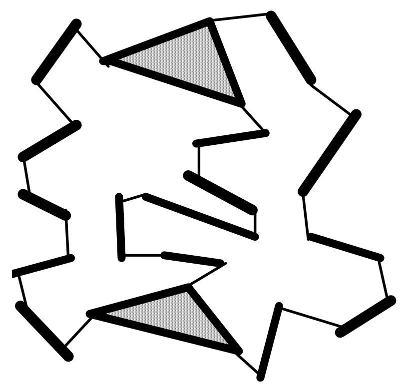


Figure 3

The third row is fulfilled by three, series, five–bar chains as in–parallel connections between two three–jointed links, Figure 3.

The fourth row gives a mechanism made of four, series, five–bar chains in–parallel between two four–jointed links, Figure. 4

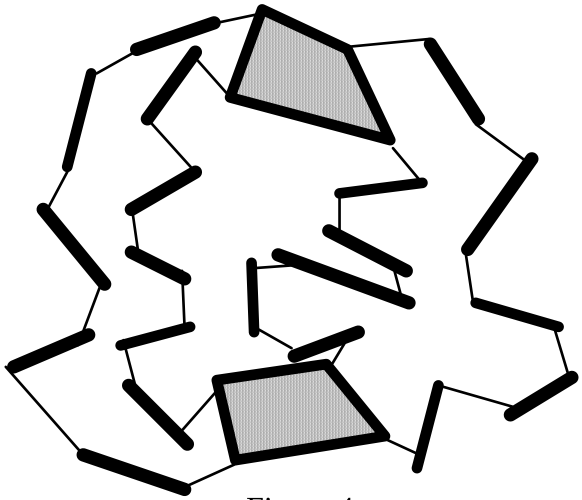


Figure 4

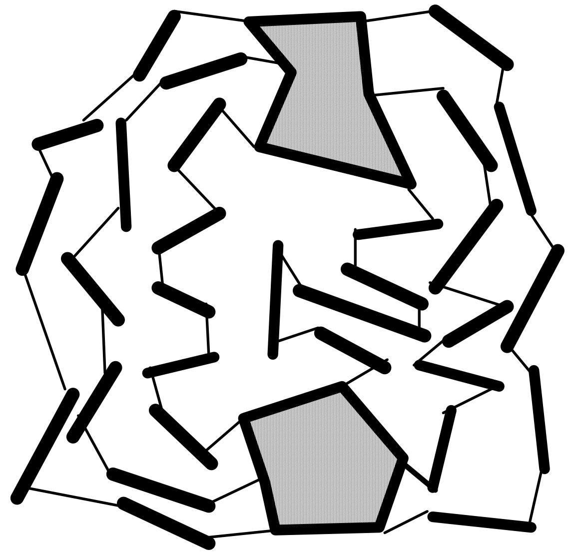


Figure. 5

Similarly, the fifth row gives a mechanism with five, series, five–bar chains as in–parallel connections between two five–jointed links. The sixth row is represented by a mechanism with six, series, five–bar chains as in–parallel connections between two six–jointed links, Figure 5. Note: if one of the five–jointed links is the ground and the other the free–end, this is the kinematic generalization of the (so–called) general Stewart Platform.

The last set of numbers given in the table represents the next step, namely: seven, series, five–bar chains between two seven–jointed links.

At this point the reader should see that we could go on adding series five–bar chains forever. For each new chain the table would be extended one row. In each new row, n is increased by simply adding 5 to n (for a new, series, five–bar chain) and 6 to p (since a series five–bar chain adds six joints). Clearly, since we have only six freedoms, once we have six driven joint, all the joints in the new chains are passive; their joint displacements are determined by the first six driven joints.

In in–parallel chains, driven joints may be allocated in various ways. For example, for the three in–parallel chains, Figure 3, we can drive two joints in each chain; or three in one, two in another and one in the third; or four in one and two in another, or one in each of the two other series chains. Finally, we could, if we wished, drive all five joints in one chain and then one joint in one of the other series chains.

In our discussion, each row in the table, other than the first, has been represented by a unique kinematic structure. However, in general, each combination of n and p represents more than one type of mechanism. In classical kinematics, graph theory has been used to determine how many different structural variants exist for each such combination. It is instructive to show the possibilities of a similar analysis in robotics.

3. LINK TYPES

Here we will simply do the first step of such an analysis; namely, determine how many joints each link can have. To do this we use the notation n_i to denote the number of links with i joints. For example, the conventional series chain six–degrees–of–freedom system shown in Figure 1a, has $n_1=2$ and $n_2=5$ (i.e., it has two links with one joint — the ground and the free–end link — and five links with two joints each.)

Clearly, $n=n_1+n_2+n_3+n_4+n_5+n_6+...$ and $p=(n_1+2n_2+3n_3+4n_4+5n_5+6n_6+\dots)/2$. If we substitute these into (2), we obtain:

$12-\frac{7}{2}n_1-n_2+\frac{3}{2}n_3+4n_4+\frac{13}{2}n_5+9n_6+\frac{23}{2}n_7+\dots=0$, or

$$12-\frac{7}{2}n_1-n_2+\frac{3}{2}n_3+4n_4+\ \dots\ +(\frac{5k}{2}-6)n_k+\ \dots\ =0 \qquad (3)$$

where k denotes the number of joints on a link. From this formula we can determine how many unitary jointed, binary jointed, ternary jointed, etc. links are contained in all possible six–degree–of–freedom linkages. (We assume multi–degree–of–freedom joints are decomposed into one–degree–of–freedom joints.)

3a. Single–jointed and two–jointed links

We approach this systematically: First, it is obvious there are no linkages with only unitary jointed links. Next, we consider linkages with only unitary and binary links therefore (3) is truncated after the n_2 term. For one degree-of-freedom joint we need to satisfy $12-\frac{7}{2}n_1-n_2=0$. Clearly $n_1=2$ and $n_2=5$ are the only positive integers to satisfy this condition. Hence there exists only one revolute–jointed linkage with only binary and unitary links: the ubiquitous open chain seven–bar shown in Figure 1a.

3b. Three–jointed links

If we now admit also ternary links, Equation (3) becomes: $12-\frac{7}{2}n_1-n_2+\frac{3}{2}n_3=0$. Which means $n_2=12-\frac{7}{2}n_1+\frac{3}{2}n_3$. For $n_3=1$, there are only two possibilities: $n_1=1$, $n_2=10$ and $n_1=3$, $n_2=3$. A value of n_1 greater than two implies a structure with connectivities all less than six, see Figure 1b for example. Hence, we have really only the first possibility, if we are interested in manipulator–type mechanisms. This is a 12 link mechanisms with one ternary, one unitary and ten binary links. There are several structural variants in how these links can be joined, the one most suitable for manipulator applications is shown in Figure 6. This configuration allows for several different choices of the base and the end–effector links.

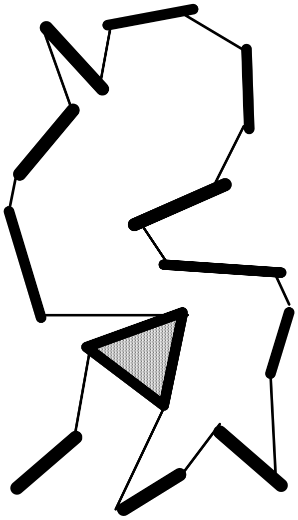


Fig. 6

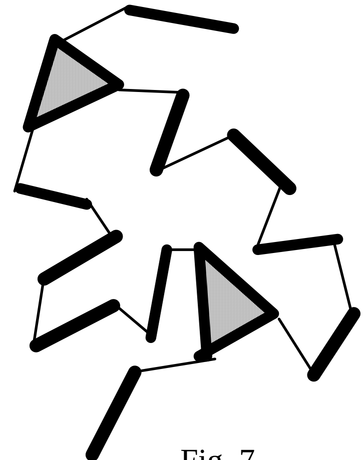


Fig. 7

For n_3=2, we find three possibilities: n_1=0, n_2=15; n_1=2, n_2=8; n_1=4, n_2=1. The first of these is the 17 link mechanism shown in Figure 3. The second is a 12 bar–mechanism with 12 joints, this is shown in Figure 7. It is interesting to compare Figure 7 to the mechanism in Figure 2 which also has twelve links and twelve joints but is composed entirely of binary links.

The third case is of no interest for six–degree–of–freedom manipulators, since n_1=4 all connectivities must be less than six. However it could be used in, for example, a four degree–of–freedom manipulator with two clamping or pushing thumbs.

In the remainder of this analysis we limit ourselves to six–degree–of–freedom manipulator type mechanisms with a single output link having connectivity six relative to a ground link, hence the condition that $n_1 \le 2$. It then follows that whenever n_3 is odd we have only one possibility, namely n_1=1, $n_2 = 12 + \frac{1}{2}(3n_3 - 7)$. This gives us one set of links for each odd value of n_3. When n_3 is even there are two possibilities for every n_3; one for n_1=0: $n_2 = 12 + \frac{3}{2} n_3$, and one for n_1=2: $n_2 = 5 + \frac{3}{2} n_3$.

3.c Four–jointed links

If we introduce the possibility of links with four joints, we have essentially the same results as in the preceding paragraphs except that n_2 is increased by 4 for each quaternary link that we introduce. The relevant formulas follow from (3): For n_3 odd, n_1=1 and $n_2 = (12 + 4n_4) + \frac{1}{2}(3n_3 - 7)$. While for n_3 even, there are two possibilities for every n_3; one for n_1=0: $n_2 = (12 + 4n_4) + \frac{3}{2} n_3$, and one for n_1=2: $n_2 = (5 + 4n_4) + \frac{3}{2} n_3$. The reader will notice that the smallest number of links which satisfy these formulas are respectively 17 (n_1=1, n_2=14, n_3=1, n_4=1), 17 (n_1=0 n_2=16, n_3=0, n_4=1) and 12 (n_1=2, n_2=9, n_3=0, n_4=1). These lead to two new variants of the n=17, p=18 group, and one new variant of the n=12, p=12 group. However only the two 17 link mechanisms, shown in Figure 8 and Figure 9a, are of possible utility for us (the 12 link mechanisms have connectivity less than six since their mobilities are distributed in three separate chains, Figure 9b.) It should be noted that in Figure 9a each closed chain has three–degrees–of–freedom, and hence the mobility is six, and the connectivity between links in the separate chains is also six. While in Figure 9b the mobility of six is divided in a one–degree–of–freedom closed chain, and a four– and one–degree–of–freedom open loop chain (which may alternatively be regarded as a single five–degree–of–freedom open chain). Hence, the connectivity is at most five.

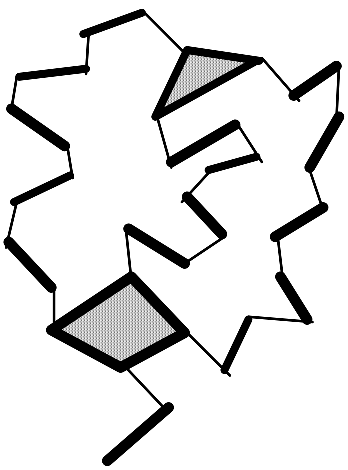


Fig. 8

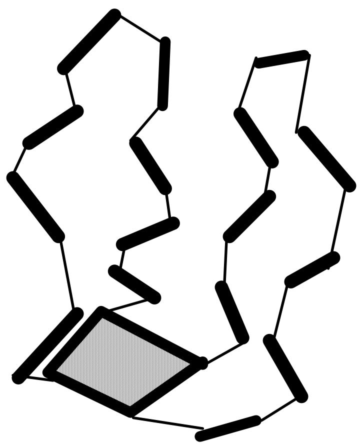


Fig. 9a

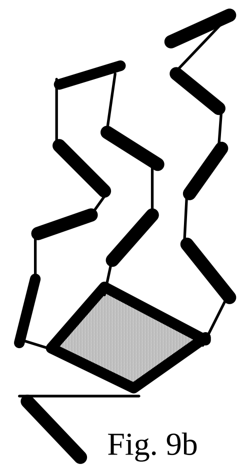


Fig. 9b

It should be noticed that the mechanism of Figure 4 has two quaternary links and twenty binary links in accordance with the above formula for n_1=0.

3d. Five–jointed links

When we have five–jointed links, the value for n_2 is governed by $n_2 = (12 + 4n_4) + \frac{1}{2}(3n_3 + +13n_5)$ when n_1=0; by $n_2 = (12 + 4n_4) + \frac{1}{2}(3n_3 + 13n_5 - 7)$ when n_1=1; and by $n_2 = (5 + 4n_4) + \frac{1}{2}(3n_3 + 13n_5)$ when n_1=2.

From these it follows that when n_5 is even, the results are the same as in the foregoing paragraphs except that the value of n_2 is increased by 13 times the number of pairs of n_5 links (i.e., for n_5 equal to 2, 4, 6, 8, etc. we increase n_2 by respectively 13, 26, 39, 52, etc.).

When n_5 is odd, the above implies that for n_1=0, n_3 must be odd. Further, when n_1=1, n_3 and n_5 must be of opposite parity, while when n_1=2, n_3 and n_5 must have the same parity. We note that the smallest number of links which satisfy these formula are respectively 22 (n_1=0, n_2=20, n_3=1, n_4=0, n_5=1), 17 (n_1=1, n_2=15, n_3=0,

n_4=0, n_5=1) and 17 (n_1=2, n_2=13, n_3=1, n_4=0, n_5=1). These lead to, respectively, one new variant of the n=22, p=24 group, and two new variants of the n=17, p=18 group. These three do not seem of value to us since in each case the maximum connectivity is less than six.

Finally, we point out that the linkage of Figure 4, belongs to this group, as given by the first formula of this section, with n_1=0, n_2=25, n_3=0, n_4=0, n_5=2.

3e. Six–jointed links

When we have links with six joints, we have the formulas:

$n_2 = (12 + 4n_4 + 9n_6) + \frac{1}{2}(3n_3 + 13n_5)$ when n_1=0; $n_2 = (12 + 4n_4 + 9n_6) + \frac{1}{2}(3n_3 + 13n_5 - 7)$ if n_1=1; and $n_2 = (5 + 4n_4 + 9n_6) + \frac{1}{2}(3n_3 + 13n_5)$ if n_1=2.

So the results are as described for the previous cases except that now n_2 is increased by 9 for each six–jointed link. We note that the minimum number of links for each formula yield, respectively, members of the n=22, p=24; n=22, p=24 and n=17, p=18 group. All of these have maximum connectivity less than 6, so they are of no interest to us. However when we go to more complex arrangements we get useful devices. For example, from the first formula when n_6=2, n_1=0, n_3=0, n_4=0, and n_5=0, we obtain n_2=30, which is the mechanism shown in Figure 5.

3f. More than six joints

We could continue the foregoing type of discussion for the groups with n_7, n_8, n_9, etc. But by now the method is clear, and both the authors and readers have had enough of these tedious details. Suffice it to say that every robotic mechanism used or yet to be designed must fall under the scope of this type of mobility analysis. If we are interested in spatial mechanisms which have mobility less (or more) than six, then the foregoing analyses and formulas are still basically valid. The only difference is that numbers 12 and 5 which appear as the first term in the formulas for n_2 need to be reduced (or increased) by the amount that the desired mobility differs from 6. So, for example, for mobility–four systems we would use 10 and 3 instead of 12 and 5. Similarly, this entire discussion could be repeated for spherical and planar mechanisms where the number 6 in (1) is replaced by 3.

4. EQUIVALENT JOINTS

Since the table goes on without limit, and for each entry there are multiple structures, we clearly have many possible mechanisms available to the robotics designer. However, to date only a small fraction of the available possibilities have been explored. One apparent difficulty is that many of the theoretically possible mechanisms seem to require a large number of links. But this is deceptive, because we have limited our discussion to one–degree–of–freedom joints. Introducing multi–degree–of–freedom joints greatly reduces the number of links and joints. This is easily done as follows.

Every multi–degree–of–freedom joint can be considered to be equivalent to a series of one–degree–of–freedom joints with special geometry. For example, a spherical joint is equivalent to three revolute joints (with concurrent axes) and two links. So by using a spherical joint we can reduce the number of links by two and the number of joints by two, and still have the same system mobility we had with revolute joints. Similarly using a two–degree–of–freedom joint reduces the number of joints and links by one, while a four–degree–of–freedom joint reduces the number of joints and links by three, and a five–degree–of–freedom joint reduces the number of links and joints by four. It is difficult to drive a true multi–degree–of–freedom joint. However this will not pose a problem if it is used as a passive joint.

To illustrate: if we consider the series five–bar chains used as in–parallel connections in the mechanisms shown in Figures 3 through 7, we could replace the first three joints by a spherical joint, and the last two joints could be replaced by a two–degree–of–freedom joint, say a Hooke’s joint. Each, series, five–bar chain is now a, series, two–bar chain; we have reduced both the number of links and number of joints in each chain by three. If we apply this reasoning to the mechanism with six such in–parallel chains, Figure 5, n and p are reduced by 18 each, and the mechanism is an $n=14$ and $p=18$ instead of an $n=32$ and $p=36$.

One caution: if the one remaining revolute joint is then replaced by a prismatic joint, the entire in–parallel chain is sometimes called a (single) actuator link. Although this description advantageously reduces n and p by an additional six each, to $n=8$ and $p=12$, it has no basis in terms of the mobility analysis derived from formula (1).

To summarize, the results derivable from (1) provide the robot designer with a large number of possible mechanisms. For each mechanism given in the table, there can be multiple structural variants, and for each structural variant there can be inversions (an inversion is where the same structure is used, but a different member is considered the base link.). Each design requires, however, that the connectivity between at least two links should be the same as the designed robot's degrees-of-freedom. The next part of this paper provides an algorithm for identifying connectivity in mechanisms.

5. CONNECTIVITY IN MECHANISMS

In mechanism theory, the term connectivity is defined as the number of degrees-of-freedom between two given links of a mechanism. Connectivity, and not the mobility, determines the ability of an output link to perform a task relative to a frame. A six-degrees-of-freedom manipulator, for example, which is able to locate an object in an arbitrary position and orientation, is actually a mechanism that has at least two links with mutual connectivity six – one of these links being the base and the other the end-effector.

As a tool to identify the connectivity between links, we utilize, in this investigation, graph theory. Graph theory has previously been applied by many investigators to identify mechanisms and detecting isomorphism [Yan and Hall, 1982; Mruthyunjaya and Balasubramanian, 1987; Rao and Varda Raju, 1991; Warnaar et al., 1992a; Rao and Rao, 1993; Tang and Liu, 1993; Jin-Kui and Wei-Qing, 1994], classify and enumerate mechanisms according to certain topological structures [Mayourian and Freudenstein, 1984; Dijksman and Timmermans, 1994; Sakamoto et. al., 1993; Liu and Yu, 1995], and automatically generate mechanisms [Sohn and Freudenstein, 1986; Sakamoto et. al., 1993]. (These citations cover only a few of the numerous investigations in this field).

The purpose of this part of the paper is to develop a simple algorithm for the derivation of the connectivity matrix of a mechanism — a subject that has not received much attention in the literature. The loop connectivity matrix, derived by Agrawal and Rao [1987] and modified slightly by Liu and Yu [1995] is different from the connectivity matrix developed in this paper: the elements of the loop connectivity matrices specify the number of common joints between each pair of loops in the mechanism, while we are interested in the mutual connectivity of any pair of links.

One note is in order. The definition of connectivity in graph theory is different from that in mechanisms. In graph theory connectivity means the number of cuts needed to separate a given connected graph.

6. APPLICATION OF GRAPH THEORY TO MEASURE CONNECTIVITY

While deriving the connectivity matrix we relied upon the well developed tools available in graph theory and attempted to embed the physical properties of a mechanism in its graph representation. It is well known that a mechanism can be uniquely represented as a graph where each link is a vertex and each joint is an edge.

For our purpose, which is measuring connectivity in a mechanism, the distance between vertices (or number of walks) — a well know term in graph theory [Biggs, 1974; Marshall, 1971] can be used.

Assume an *open kinematic chain* with each joint having one degree-of-freedom. Then, the number of joints between a pair of links, namely their connectivity, equals the distance between the corresponding vertices in the mechanism's graph representation. The same concept is true for joints having more than one degree-of-freedom provided, that in this case, the weighted distance is used. For *closed kinematic chains* this observation does not hold and the distance between vertices is not a measure of their connectivity.

Since the mathematical tool and algorithm of graph theory are well developed, it is appealing to use these tools to analyze the connectivity of general mechanisms — both open and closed kinematic chains. The question therefore is: what changes should be made in a graph representation of a mechanism to embed the properties of a closed kinematic chain in its graph so as to make the analysis of the connectivity possible in the same simple way as it is for an open chain.

Before proper changes in the graph can be made, it is necessary to first examine the differences between graphs of closed and open kinematic chains in respect to connectivity.

These differences are two–fold: First, the graph representation of a closed kinematic chain is, from the graph theory point of view, no longer a tree since it contains loops. Hence, there is more than one path between vertices and, consequently, more than a single distance. Secondly, a loop in a kinematic chain reduces the mobility of the mechanism. (Recall that the mobility of the mechanism must be an upper bound on all connectivities in the mechanism.) A loop may have mobility zero, such as in a spatial loop with six or less (arbitrarily situated) one–degree–of–freedom revolute joints, and thus reduce the loop to a structure. For a loop with more than six one–degree–of–freedom joints, the mobility of an isolated loop is equal to the number of joints minus six (minus three in the planar or spherical cases).

It needs to be mentioned that no geometrical relationship between joints is assumed to exist, so the neighboring joints axes can be constructed to have any relative direction in space. This joint arrangement constitutes, in the spatial case, a screw system of order six and, in the spherical and planar cases, screw system of order three [Hunt, 1983]. If, however, geometrical relations between joints exists, such as parallelism or intersection, then the loop might posses additional mobility (and different connectivity).

7. DERIVATION OF THE CONNECTIVITY MATRIX

Starting our analysis with open kinematic chain mechanisms, the connectivity, c_{ij}, between links i and j is measured as the weighted distance between vertices i and j in the mechanism's graph representation. Hence :

$$c_{ij} = d_{ij} \tag{4}$$

where d_{ij} is the weighted-distance between vertices i and j. We use the word distance, in the sequel, to denote weighted-distance.

In order to use the distance as a measurement of connectivity even in the presence of loops, one has to modify the mechanism's graph representation so that it captures the features of a closed–loop mechanism. Therefore, the following steps are suggested:

1. Since each loop which is a structure is in effect a single link, such a loop is shrunk to a single vertex in the graph (this idea is mentioned also in Warnaar et al. [1992b]).

2. Virtual edges (joints) are added so that the same procedure used for open kinematic chains, namely taking the distance between vertices as the measure of connectivity, is applicable also for a general mechanism. The procedure for adding these edges and its proof of validity are described below.

Assuming a one-loop mechanism, k, its connectivity is upper bounded by the loop mobility, hence the connectivity within the loop is given by :

$$c_{ij} = \min\left(d_{ij}, M_k\right) \tag{5}$$

where d_{ij} is the shortest distance between vertices (links) i and j, and M_k is the loop mobility given by:

$$M_k = \sum_{i=1}^{p} f_i - 6 \tag{6}$$

Next, one has to determine the cases where mobility, and not the distance, determines the connectivity, in other words, when the loop mobility is less than the distance. Observing that since any two links are connected by two different sides of the chain, and that the shortest of these two distances determines their connectivity, the distance is bounded by :

$$d_{ij} \leq \frac{1}{2}\sum_{i=1}^{p} f_i \tag{7}$$

The connectivity is, therefore, limited by the loop mobility and not by the distance only when

$$\left(\sum_{i=1}^{p} f_i - 6\right) < \mathrm{int}\left(\frac{1}{2}\sum_{i=1}^{p} f_i\right). \tag{8}$$

Since we are not interested in cases where the sum of the joint freedom is less than 6, this inequality yields

$$6 < \sum_{i=1}^{p} f_i < 11 \tag{9}$$

which implies that the only loops where mobility, and not the distance (weighted number of joints between links), determines the connectivity are those having joint–freedom sums of 7, 8, 9 or 10.

8. ADDING VIRTUAL EDGES

Our goal to have the connectivity of a general mechanism determined by the distance between vertices of its graph, calls for a change in the graph representation of mechanisms with loops which have joint–freedom sums of 7, 8, 9 or 10.

For those cases, where connectivity is determined by loop mobility rather than by distance between vertices, one can add weighted virtual edges so that the distances is bounded by the mobility of the loop. Adding these edges, the original and the resulting graphs of these loops are shown in Figure 10, where the added edges have weights 1, 2, 3, and 4 in the 7, 8, 9, and l0–jointed loops respectively.

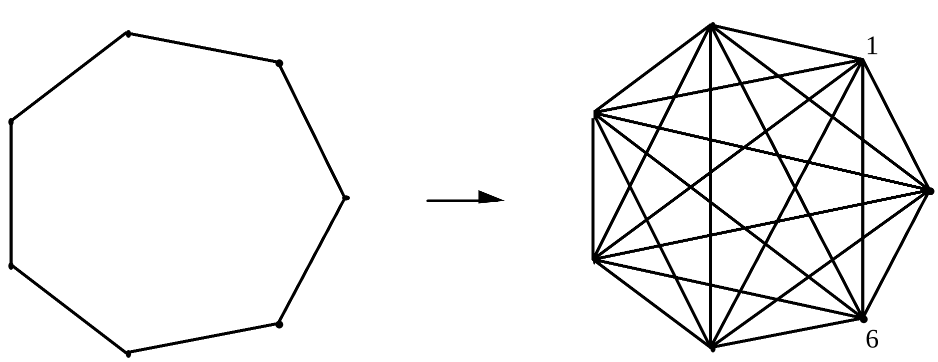


Figure 10a. All the added edges have a weight of one.

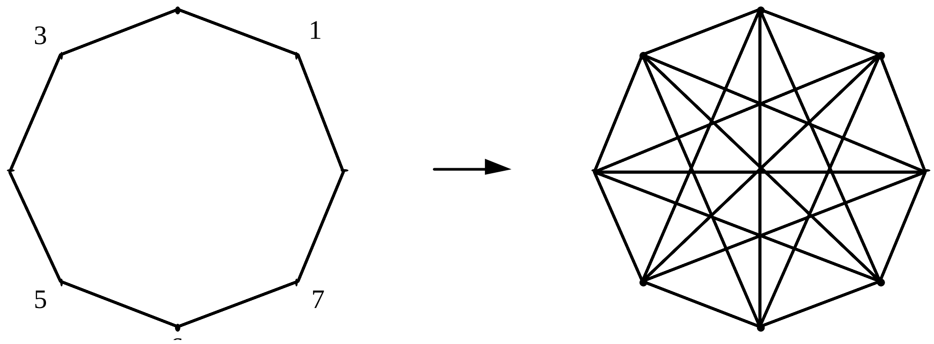


Figure 10b. All the added edges have a weight of two.

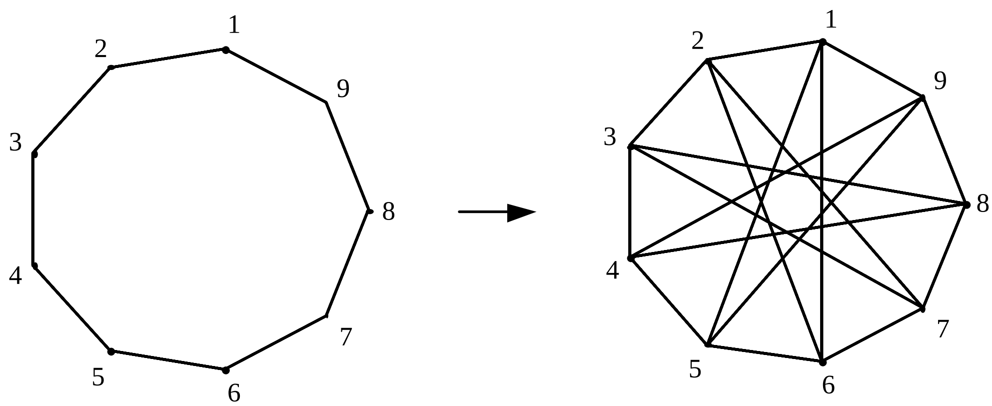


Figure 10c. All the added edges have a weight of three.

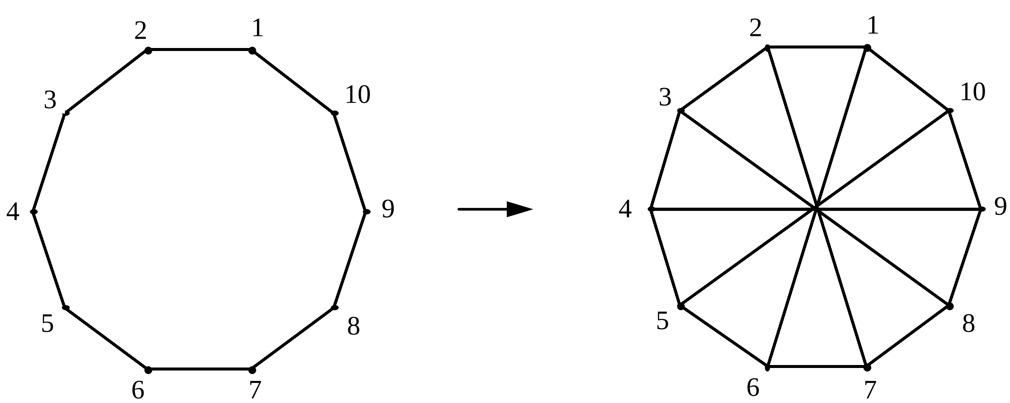


Figure 10d. All the added edges have a weight of four.

Figure 10 Adding virtual edges for spatial loops

Following the same procedure for the planar case, it turns out that there is only one case which requires modification: the four link, one-degree-of-freedom loop. This is shown in Figure 11.

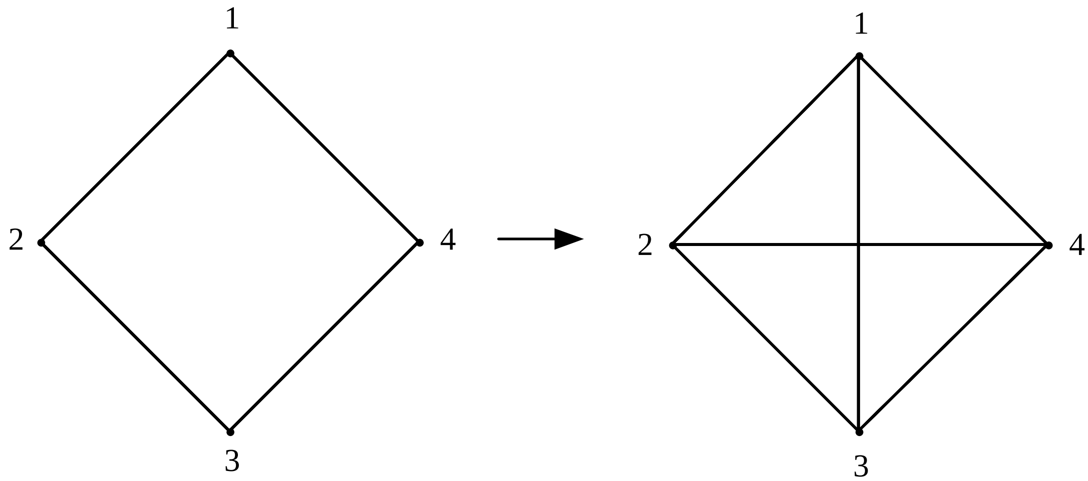
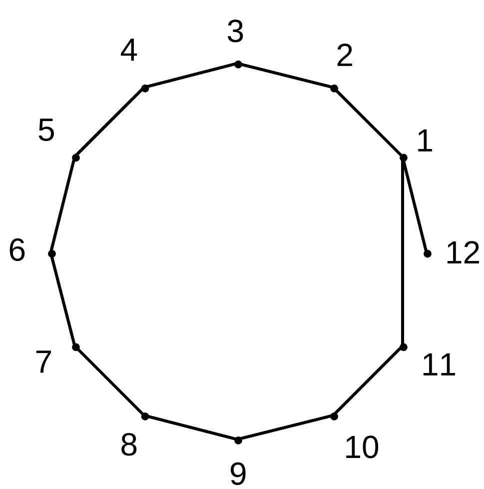


Figure 11 Adding virtual edges, of weight one, in a planar loop

9. APPLICATION OF THE CONNECTIVITY MATRIX

Once the above algorithm is applied to the shortest independent loops (or, as it termed in graph theory, fundamental cycles [Foulds, 1992]) the connectivity matrix of the mechanism is obtained simply as the distance between vertices .

As an example consider the mechanisms shown in Figure 6, with its graph representation shown in Figure 12. Since its loop contains more than 10 edges, no change is required in the graph, and the connectivity is given by the following distance matrix:

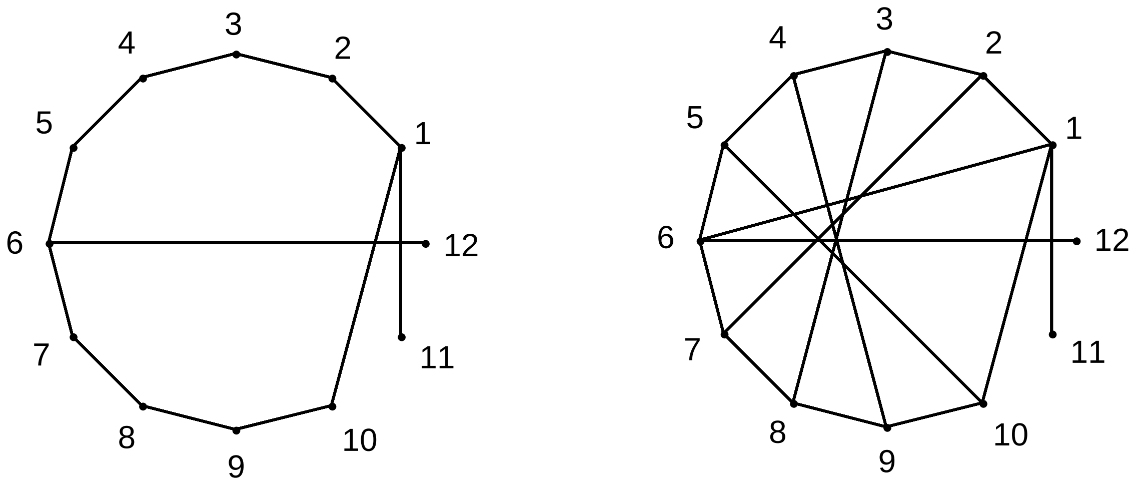


$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 5 & 4 & 3 & 2 & 1 & 1 \\ 1 & 0 & 1 & 2 & 3 & 4 & 5 & 5 & 4 & 3 & 2 & 2 \\ 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 5 & 4 & 3 & 3 \\ 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 5 & 4 & 4 \\ 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 5 & 5 \\ 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 6 \\ 4 & 5 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 5 \\ 3 & 4 & 5 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 4 \\ 2 & 3 & 4 & 5 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 3 \\ 1 & 2 & 3 & 4 & 5 & 5 & 4 & 3 & 2 & 1 & 0 & 2 \\ 1 & 2 & 3 & 4 & 5 & 6 & 6 & 5 & 4 & 3 & 2 & 0 \end{bmatrix}$$

Figure 12 Graph representation and connectivity matrix of the mechanism shown in Figure 6

As can be seen from the above matrix, the only pairs with connectivity six are links 6 and 12 and links 7 and 12.

The graph representation of the mechanism in Figure 7 needs to be modified, Figure 13, since it contains a loop with 10 one-degree-of-freedom joints .



Unmodified graph Modified graph

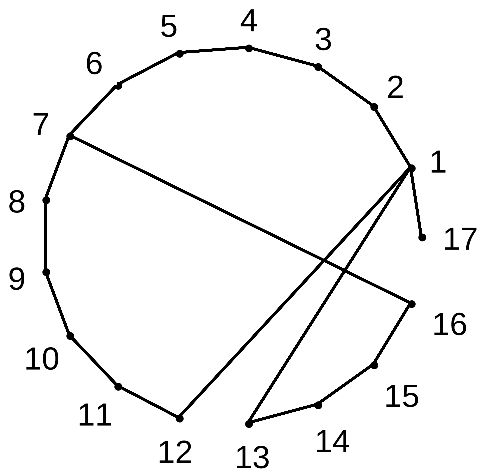
Figure 13 Graph representation of the mechanism shown in Figure 7

Remembering that the added edges have a weight of four, the connectivity is given by the distance matrix:

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 4 & 4 & 3 & 2 & 1 & 1 & 5 \\ 1 & 0 & 1 & 2 & 3 & 4 & 4 & 4 & 3 & 2 & 2 & 5 \\ 2 & 1 & 0 & 1 & 2 & 3 & 4 & 4 & 4 & 3 & 3 & 4 \\ 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 4 & 4 & 4 & 3 \\ 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 4 & 5 & 2 \\ 4 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 1 \\ 4 & 4 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 5 & 2 \\ 3 & 4 & 4 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 4 & 3 \\ 2 & 3 & 4 & 4 & 4 & 3 & 2 & 1 & 0 & 1 & 3 & 4 \\ 1 & 2 & 3 & 4 & 4 & 4 & 3 & 2 & 1 & 0 & 2 & 5 \\ 1 & 2 & 3 & 4 & 5 & 5 & 5 & 4 & 3 & 2 & 0 & 6 \\ 5 & 5 & 4 & 3 & 2 & 1 & 2 & 3 & 4 & 5 & 6 & 0 \end{bmatrix}$$

Which shows that there is only one pair with connectivity six: links 11 and 12.

For the mechanism shown in Figure 8, no modification is required and its graph and connectivity matrix are:

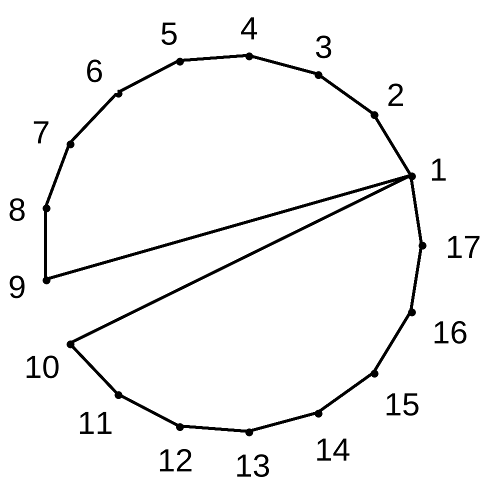


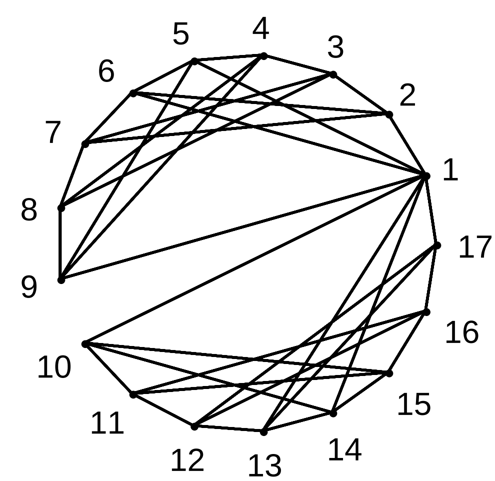
$$\begin{bmatrix}
0 & 1 & 2 & 3 & 4 & 5 & 5 & 5 & 4 & 3 & 2 & 1 & 1 & 2 & 3 & 4 & 1 \\
1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 2 & 3 & 4 & 5 & 2 \\
2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 3 & 4 & 5 & 5 & 3 \\
3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 4 & 5 & 5 & 4 & 4 \\
4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 5 & 5 & 4 & 3 & 5 \\
5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 6 \\
5 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 4 & 3 & 2 & 1 & 6 \\
5 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 4 & 3 & 2 & 6 \\
4 & 5 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 5 & 5 & 4 & 3 & 5 \\
3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 4 & 5 & 5 & 4 & 4 \\
2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 3 & 4 & 5 & 5 & 3 \\
1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 2 & 3 & 4 & 5 & 2 \\
1 & 2 & 3 & 4 & 5 & 5 & 4 & 5 & 5 & 4 & 3 & 2 & 0 & 1 & 2 & 3 & 2 \\
2 & 3 & 4 & 5 & 5 & 4 & 3 & 4 & 5 & 5 & 4 & 3 & 1 & 0 & 1 & 2 & 3 \\
3 & 4 & 5 & 5 & 4 & 3 & 2 & 3 & 4 & 5 & 5 & 4 & 2 & 1 & 0 & 1 & 4 \\
4 & 5 & 5 & 4 & 3 & 2 & 1 & 2 & 3 & 4 & 5 & 5 & 3 & 2 & 1 & 0 & 5 \\
1 & 2 & 3 & 4 & 5 & 6 & 6 & 6 & 5 & 4 & 3 & 2 & 2 & 3 & 4 & 5 & 0
\end{bmatrix}$$

Figure 14 Graph representation and connectivity matrix of the mechanism shown in Figure 8

This mechanism contains eight pairs of links with connectivity six.

The graph representation of the mechanism in Figure 9a needs to be modified since it contains two 9–link loops. The modified graph and its connectivity matrix are shown below:





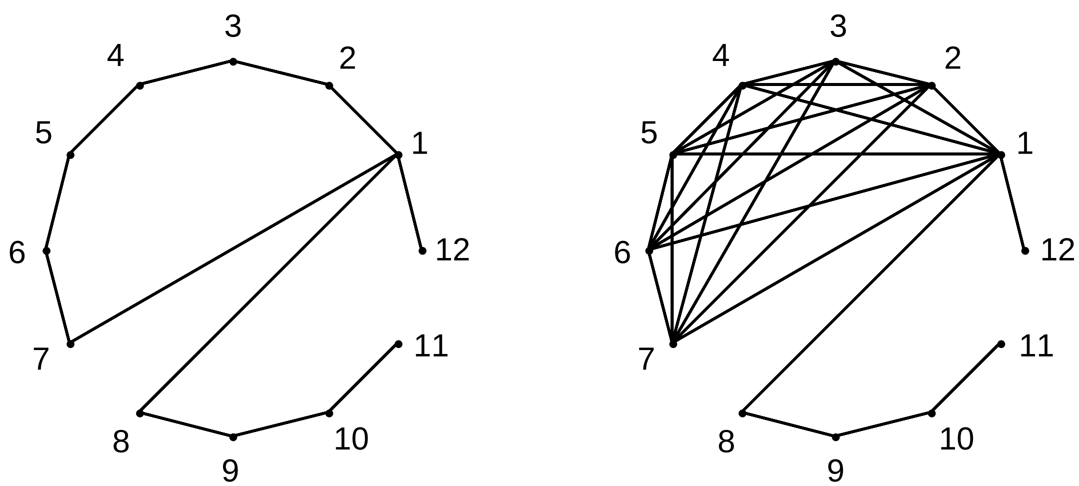
Unmodified graph Modified graph with added edges of weight three

$$
\begin{bmatrix}
0 & 1 & 2 & 3 & 3 & 3 & 3 & 2 & 1 & 1 & 2 & 3 & 3 & 3 & 3 & 2 & 1 \\
1 & 0 & 1 & 2 & 3 & 3 & 3 & 3 & 2 & 2 & 3 & 4 & 4 & 4 & 4 & 3 & 2 \\
2 & 1 & 0 & 1 & 2 & 3 & 3 & 3 & 3 & 3 & 4 & 5 & 5 & 5 & 5 & 4 & 3 \\
3 & 2 & 1 & 0 & 1 & 2 & 3 & 3 & 3 & 4 & 5 & 6 & 6 & 6 & 6 & 5 & 4 \\
3 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 3 & 4 & 5 & 6 & 6 & 6 & 6 & 5 & 4 \\
3 & 3 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 6 & 6 & 6 & 5 & 4 \\
3 & 3 & 3 & 3 & 2 & 1 & 0 & 1 & 2 & 4 & 5 & 6 & 6 & 6 & 6 & 5 & 4 \\
2 & 3 & 3 & 3 & 3 & 2 & 1 & 0 & 1 & 3 & 4 & 5 & 5 & 5 & 5 & 4 & 3 \\
1 & 2 & 3 & 3 & 3 & 3 & 2 & 1 & 0 & 2 & 3 & 4 & 4 & 4 & 4 & 3 & 2 \\
1 & 2 & 3 & 4 & 4 & 4 & 4 & 3 & 2 & 0 & 1 & 2 & 3 & 3 & 3 & 3 & 2 \\
2 & 3 & 4 & 5 & 5 & 5 & 5 & 4 & 3 & 1 & 0 & 1 & 2 & 3 & 3 & 3 & 3 \\
3 & 4 & 5 & 6 & 6 & 6 & 6 & 5 & 4 & 2 & 1 & 0 & 1 & 2 & 3 & 3 & 3 \\
3 & 4 & 5 & 6 & 6 & 6 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 3 \\
3 & 4 & 5 & 6 & 6 & 6 & 6 & 5 & 4 & 3 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\
3 & 4 & 5 & 6 & 6 & 6 & 6 & 5 & 4 & 3 & 3 & 3 & 2 & 1 & 0 & 1 & 2 \\
2 & 3 & 4 & 5 & 5 & 5 & 5 & 4 & 3 & 3 & 3 & 3 & 3 & 2 & 1 & 0 & 1 \\
1 & 2 & 3 & 4 & 4 & 4 & 4 & 3 & 2 & 2 & 3 & 3 & 3 & 3 & 2 & 1 & 0
\end{bmatrix}
$$

Figure 15 Graph representations and connectivity matrix of the mechanism shown in Figure 9a

As many as 16 pairs of links with connectivity six are in this mechanism.

It is easy to see that the mechanism shown in Figure 9b does not contain any pair of links having connectivity six. This is born out by its connectivity matrix as shown below.

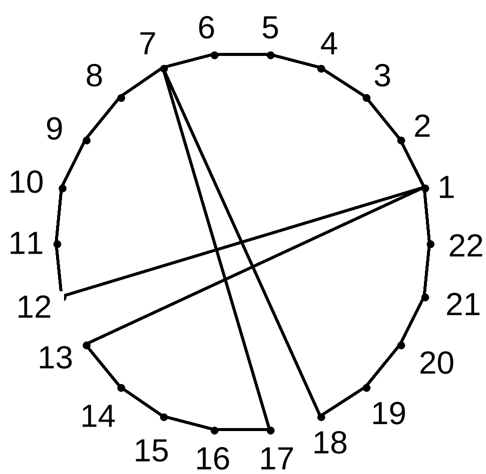


Unmodified graph   Modified graph where added edges have a weight of one

$$
\begin{bmatrix}
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 3 & 4 & 1 \\
1 & 0 & 1 & 1 & 1 & 1 & 1 & 2 & 3 & 4 & 5 & 2 \\
1 & 1 & 0 & 1 & 1 & 1 & 1 & 2 & 3 & 4 & 5 & 2 \\
1 & 1 & 1 & 0 & 1 & 1 & 1 & 2 & 3 & 4 & 5 & 2 \\
1 & 1 & 1 & 1 & 0 & 1 & 1 & 2 & 3 & 4 & 5 & 2 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 2 & 3 & 4 & 5 & 2 \\
1 & 2 & 2 & 2 & 2 & 2 & 2 & 0 & 1 & 2 & 3 & 2 \\
2 & 3 & 3 & 3 & 3 & 3 & 3 & 1 & 0 & 1 & 2 & 3 \\
3 & 4 & 4 & 4 & 4 & 4 & 4 & 2 & 1 & 0 & 1 & 4 \\
4 & 5 & 5 & 5 & 5 & 5 & 5 & 3 & 2 & 1 & 0 & 5 \\
1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 4 & 5 & 0
\end{bmatrix}
$$

Figure 16 Graph representation and connectivity matrix of the mechanism shown in Figure 9b

The twenty-two-link mechanism shown in Figure 4 have 31 pairs of links with connectivity six (the mobility, of course, is still six). This follows from the following graph and matrix.



$$
\begin{bmatrix}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 1 & 1 & 2 & 3 & 4 & 5 & 5 & 4 & 3 & 2 & 1 \\
1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 2 & 3 & 4 & 5 & 6 & 6 & 5 & 4 & 3 & 2 \\
2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 3 & 4 & 5 & 6 & 5 & 5 & 6 & 5 & 4 & 3 \\
3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 4 & 5 & 6 & 5 & 4 & 4 & 5 & 6 & 5 & 4 \\
4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 5 & 6 & 5 & 4 & 3 & 3 & 4 & 5 & 6 & 5 \\
5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 6 & 5 & 4 & 3 & 2 & 2 & 3 & 4 & 5 & 6 \\
6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 5 & 4 & 3 & 2 & 1 & 1 & 2 & 3 & 4 & 5 \\
5 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 6 & 5 & 4 & 3 & 2 & 2 & 3 & 4 & 5 & 6 \\
4 & 5 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 5 & 6 & 5 & 4 & 3 & 3 & 4 & 5 & 6 & 5 \\
3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 4 & 5 & 6 & 5 & 4 & 4 & 5 & 6 & 5 & 4 \\
2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 3 & 4 & 5 & 6 & 5 & 5 & 6 & 5 & 4 & 3 \\
1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 2 & 3 & 4 & 5 & 6 & 6 & 5 & 4 & 3 & 2 \\
1 & 2 & 3 & 4 & 5 & 6 & 5 & 6 & 5 & 4 & 3 & 2 & 0 & 1 & 2 & 3 & 4 & 6 & 5 & 4 & 3 & 2 \\
2 & 3 & 4 & 5 & 6 & 5 & 4 & 5 & 6 & 5 & 4 & 3 & 1 & 0 & 1 & 2 & 3 & 5 & 6 & 5 & 4 & 3 \\
3 & 4 & 5 & 6 & 5 & 4 & 3 & 4 & 5 & 6 & 5 & 4 & 2 & 1 & 0 & 1 & 2 & 4 & 5 & 6 & 5 & 4 \\
4 & 5 & 6 & 5 & 4 & 3 & 2 & 3 & 4 & 5 & 6 & 5 & 3 & 2 & 1 & 0 & 1 & 3 & 4 & 5 & 6 & 5 \\
5 & 6 & 5 & 4 & 3 & 2 & 1 & 2 & 3 & 4 & 5 & 6 & 4 & 3 & 2 & 1 & 0 & 2 & 3 & 4 & 5 & 6 \\
5 & 6 & 5 & 4 & 3 & 2 & 1 & 2 & 3 & 4 & 5 & 6 & 6 & 5 & 4 & 3 & 2 & 0 & 1 & 2 & 3 & 4 \\
4 & 5 & 6 & 5 & 4 & 3 & 2 & 3 & 4 & 5 & 6 & 5 & 5 & 6 & 5 & 4 & 3 & 1 & 0 & 1 & 2 & 3 \\
3 & 4 & 5 & 6 & 5 & 4 & 3 & 4 & 5 & 6 & 5 & 4 & 4 & 5 & 6 & 5 & 4 & 2 & 1 & 0 & 1 & 2 \\
2 & 3 & 4 & 5 & 6 & 5 & 4 & 5 & 6 & 5 & 4 & 3 & 3 & 4 & 5 & 6 & 5 & 3 & 2 & 1 & 0 & 1 \\
1 & 2 & 3 & 4 & 5 & 6 & 5 & 6 & 5 & 4 & 3 & 2 & 2 & 3 & 4 & 5 & 6 & 4 & 3 & 2 & 1 & 0
\end{bmatrix}
$$

Figure 16 Graph representation and connectivity matrix of the mechanism shown in Figure 4

10. CONCLUSIONS

The connectivity between two given links determines their relative freedom. For many mechanisms the link connectivity differs from link to link, and is different than the mobility of the mechanism. Hence, if we say a manipulator has six degrees–of–freedom, we actually mean it has connectivity six between its base and its end–effector.

In this paper, staring with the mobility equation, we determined all possible combinations of links, with two, three, four, five and six-jointed links, that result in a "mobility six" mechanism. Since the connectivity and not the mobility defines the relative degree-of-freedom between two links, a connectivity matrix was developed where each element a_{ij} gives the relative connectivity between links i and j. Utilizing graph theory, it was shown that the connectivity of a mechanism, whether a closed or an open kinematic chain, can be obtained by the well known distance (walks) algorithm of its graph. To use this algorithm for graphs having loops (closed kinematic chains) it is necessary to modify the graphs so that the kinematic characteristics of the mechanism are captured. To achieve this goal, each structure has to be shrunk into a vertex (one link) and four types of graphs, those representing loops with joint freedoms summing to 7, 8, 9, or 10, have to be modified by the addition of weighted virtual edges. For planar and spherical mechanisms only the four–degree–of–freedom mechanisms' graphs needs to be modified.

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