Comments are invited and should be directed to the authors at the address listed below.
Please do not reproduce in any way without the permission of the authors.

Assembly Mode Changing
in
Parallel Mechanisms:
Rising on a Ramp

by

H. Bamberger, A. Wolf, M. Shoham

TME Report – 482

July 2007

Robotics Laboratory
Faculty of Mechanical Engineering
Technion - Israel Institute of Technology
Haifa, Israel 32000
Email: hagayb@technion.ac.il; alonw@technion.ac.il; shoham@technion.ac.il

Abstract

Parallel mechanisms usually have several direct kinematic solutions that are attributed to different assembly modes or postures. It is well known that changing assembly mode without crossing any singularity is possible by encircling a cusp point. This report describes an assembly-mode changing, which is not caused by encircling any cusp point, but by moving on a path like a ramp in a 3D space, that describes the assembly modes above the joint-space.

Introduction

Changing assembly modes, where the same joint coordinates result in different outputs, was long perceived as being connected to crossing singular configurations. While this is the case in most existing robots, it is incorrect in the general case [1]. Recently, singularity-free assembly mode changing was reported and explained by geometric means [1]–[6]. This phenomenon is important both from a control and kinematic point of view, since a smooth movement between assembly modes is doable, and, on the other hand, undesirable assembly mode changes may occur. On the other hand, geometrical conditions were found where assembly mode changing is impossible unless one of the mechanism joints is physically disconnected [7]-[8].

Singularities are usually determined by observing the Jacobian matrices, which relate the input joint rates to the platform output velocity [9]–[10]. This relationship is given by $\mathbf{J}_x \dot{\mathbf{x}} = \mathbf{J}_q \dot{\mathbf{q}}$, where $\mathbf{q}$ is the vector of the actuated joint variables, and $\mathbf{x}$ is the displacement vector of the moving platform. An inverse kinematic singularity occurs when the determinant of the Jacobian $\mathbf{J}_q$ vanishes, namely, det($\mathbf{J}_q$)=0. In this case, the mechanism loses one or more degrees of freedom [9]–[11]. This type of singularity is similar to that of a serial mechanism, and it always occurs at the workspace boundary, where two inverse kinematic solutions converge.

On the other hand, when det($\mathbf{J}_x$)=0, a direct kinematic singularity occurs, and the mechanism gains one or more uncontrollable degrees of freedom [9]–[11]. Nonzero $\dot{\mathbf{x}}$ vectors may result in $\dot{\mathbf{q}} = 0$, meaning that the moving platform may possess infinitesimal motion while all the actuators are completely locked. Direct kinematic singularities always occur in configurations where different branches of the direct kinematic solutions meet. Those branches are called the assembly modes. The term "assembly mode" (AM) is used for branches of direct kinematics but not for branches of inverse kinematics. Passing through a singularity of the inverse kinematic solution is not critical. In this singularity the mechanism is still controllable, thus avoiding the inverse kinematic singularity is desirable but less critical than the direct kinematic singularity [3].

Changing assembly modes was usually related to passing through direct kinematic singular configuration [11]. Recently, several researchers [3]–[5] showed a method to pass between two AMs for 3RPR parallel mechanisms, without meeting any singularity. This method is based on identifying points in the joint-space, where three direct kinematic solutions coincide. These points are called cusp points, and by encircling them on a closed path in the joint-space, one may pass between two AMs. Moreover, since these AMs are actually connected, it is difficult to discern between them [6]. It is worth noting that joint limitations and collisions between mechanism parts are neglected, although they may preclude an AM change. An intuitive explanation why this phenomenon exists is given by Macho et al. [2], who showed that only one singularity surface (where $\det(\mathbf{J}_x)=0$) divides the workspace into two regions (where $\det(\mathbf{J}_x)>0$ and $\det(\mathbf{J}_x)<0$). Hence, if the direct kinematic problem has more than two solutions, it is obvious that at least two solutions lie in the same region, meaning that non-singular path between these assembly modes may be found.

In addition, it has been shown that non-singular AM changing occurs in serial mechanisms by moving on a workspace closed path [1]. In the serial case, the cusp point is defined as a meeting between three inverse kinematic solutions, and the assembly modes are defined as branches of these solutions. Such a mechanism was called cuspidal by Baili et al. [12], which showed geometric conditions to classify a 3R mechanism by its number of cusp points. Innocenti et al. [3] dealt both with serial and parallel mechanisms, and proved that for the serial case the joint-space is divided into two regions (where $\det(\mathbf{J}_q)>0$ and $\det(\mathbf{J}_q)<0$) by only one singularity surface (where $\det(\mathbf{J}_q)=0$).

This phenomenon works also for 3PRR parallel mechanism. Another method of AM changing, which does not exist in 3RPR mechanisms, is described in this report. It should be noted that during this assembly mode changing, the mechanism passes via an inverse kinematic singularity.

Findings

The mechanism under discussion is a 3<u>P</u>RR robot, described in the following figure:

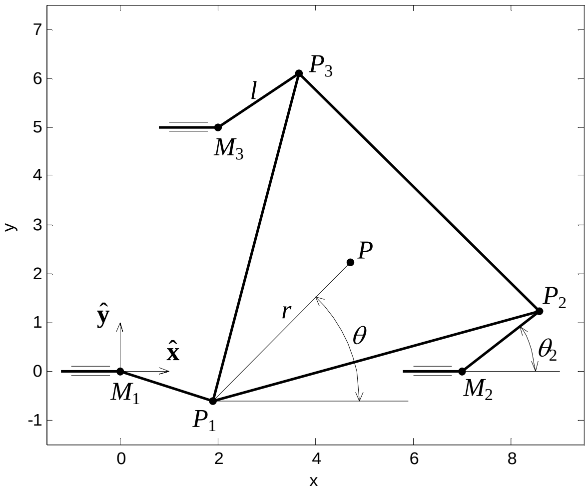

Fig. 1: The kinematical structure

The robot consists of an equilateral triangle platform, whose center is the point *P*. The platform pose is determined by point *P* *x* and *y* coordinates and by the platform orientation θ. Points P_1, P_2, and P_3 are located on the platform in an equal distance *r* from the platform center *P*. The lengths of the links which connect the actuators with the platform are *l*, and the angles between those links and *x*-axis are θ_i (i=1,2,3). The linear actuators determine the position of the points M_i. For simplicity of the mathematics, all the actuators move parallel to *x*-axis, and M_1 and M_2 share the same line of action.

The singularity map shows the combination of actuators' locations in which at least two solutions of the direct kinematics problem (DKP) merge. Actually, these situations are described by curves in the joint space, where between these curves, one can find areas of existence of 2, 4 or 6 solutions (=assembly modes).

The singularity map for M_{1x}=0 is shown in the next figure, followed by a zoom in the interesting area:

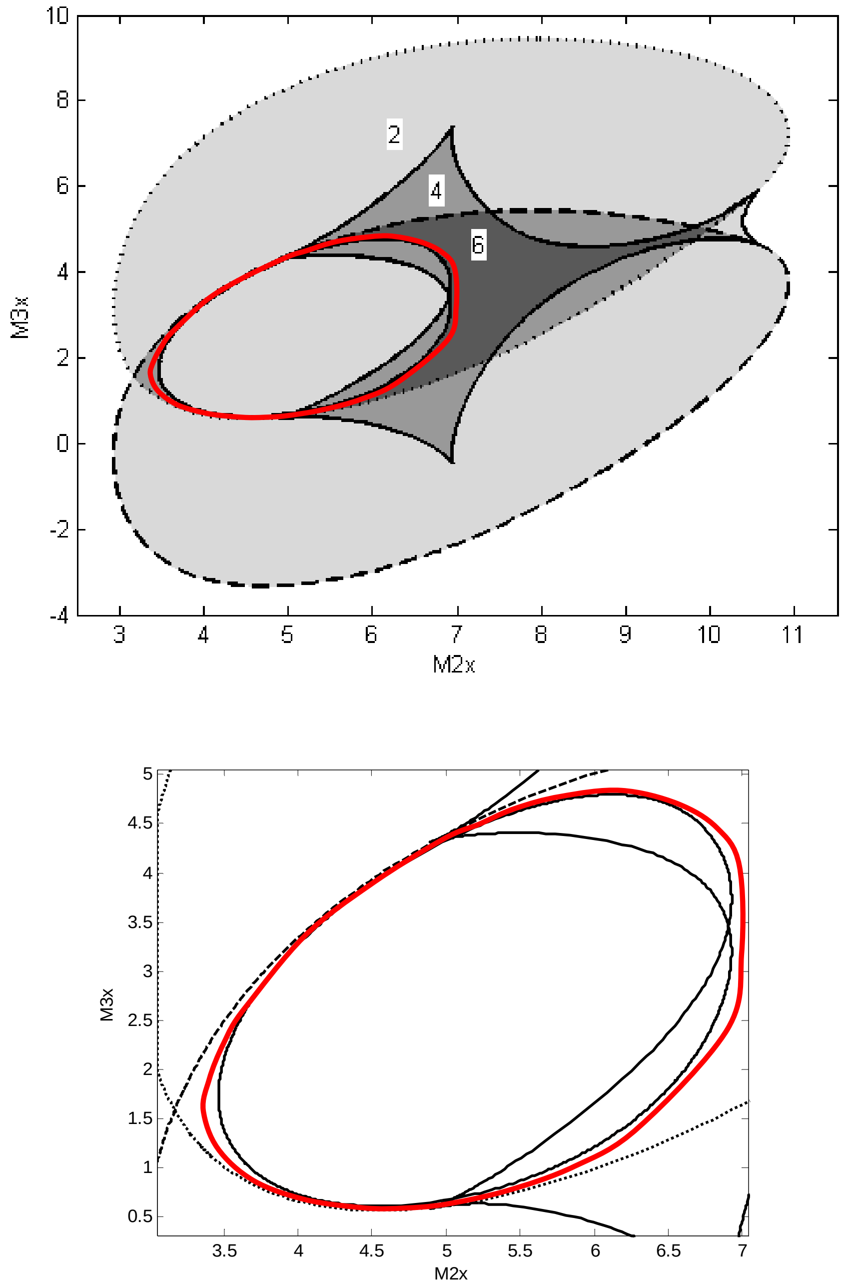

Fig. 2: The singularity map in the joint space

The phenomenon of assembly-mode changing occurs when the actuators moves along the red curve through one of the “narrow bridges”. Note that in these narrow bridges, at about (M_{2x}, M_{3x})=(4.5, 0.6) and (M_{2x}, M_{3x})=(4.4, 3.8), four solutions of the DKP exist. The phenomenon can be better understood by observing the following figures.

The singularity map can be extended to a 3D figure, where the third axis describes the platform pose, using an arbitrary parameter. Each assembly mode is a surface in this space, that its coordinates are the actuator locations and the value of that chosen parameter. The next figures show the six assembly modes in the interval $2.9 \leq M_{2x} \leq 7.3$, $-1 \leq M_{3x} \leq 5$, where the angle θ_3 appears in the third axis. The upper right figure is a projection, describing the existence areas of the assembly mode:

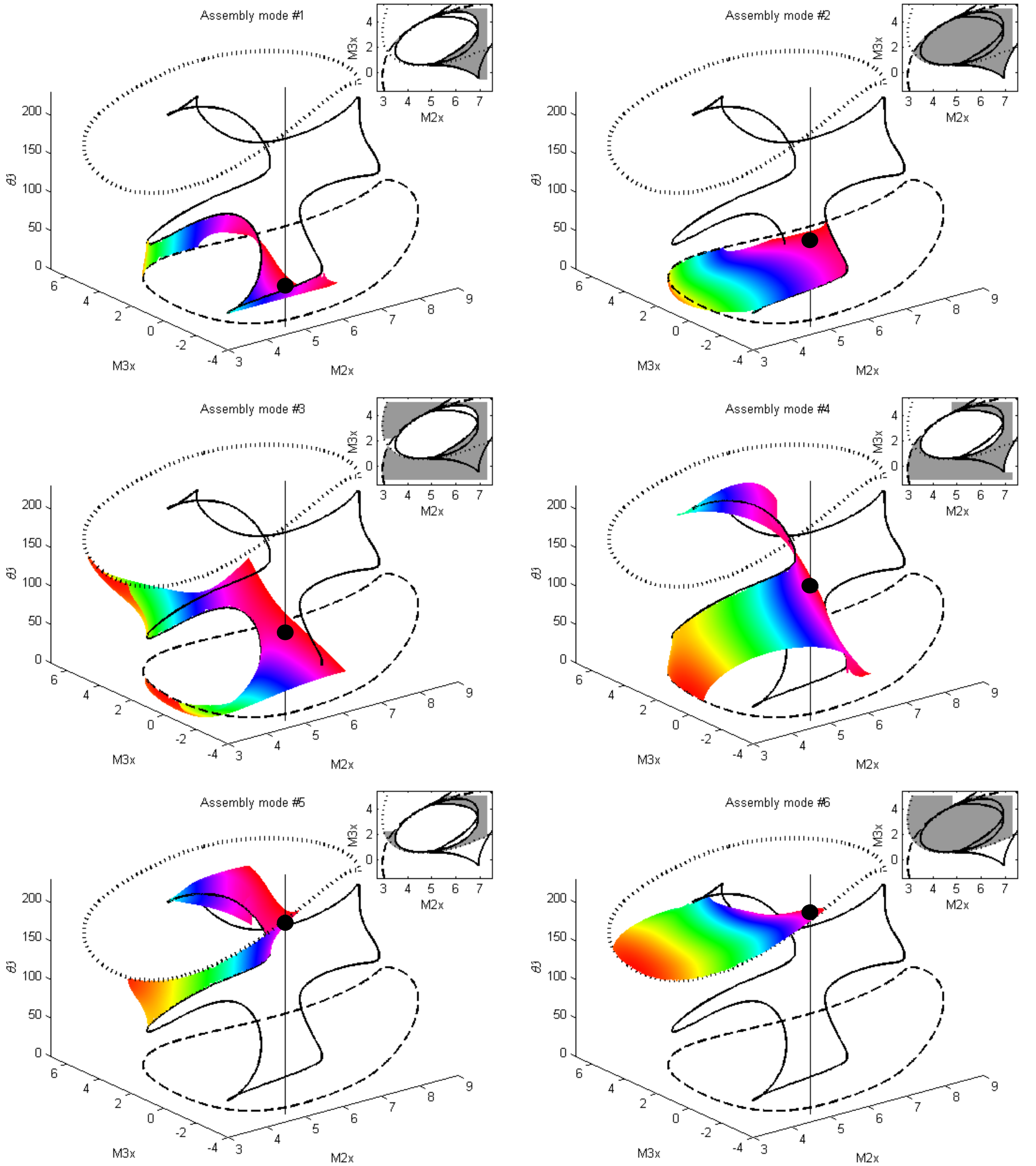

Fig. 3: The assembly modes

The numbering of the assembly modes is arbitrary, and was determined in ascending order of θ_3 at arbitrary joints values. These specific joint values are described by a vertical line in Fig. 3, where the values of θ_3 are the black dots on it.

The surfaces of the third and the fifth assembly modes are actually connected to each other, as can be seen in the next figures, where these assembly modes were drawn together:

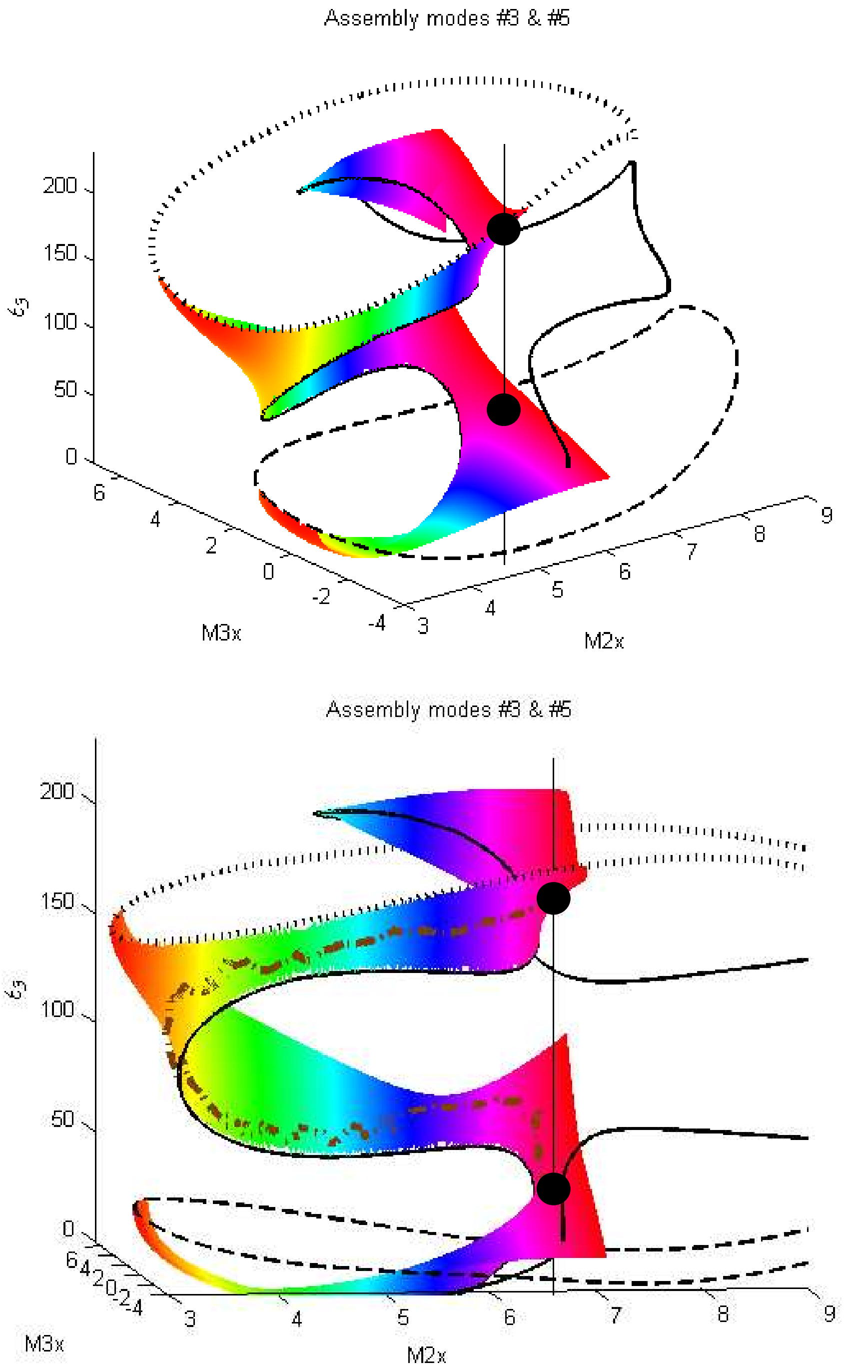

Fig. 4: The connected assembly modes #3 and #5

Therefore, one can move between the black dots by moving the actuators along the red curves of Fig. 2. **This assembly mode change is not implicated by encircling any cusp point nor by entering to a singular pose.** The motion reminds us a rising on a ramp between two floors of a parking lot, that why we call it **"ramp"**.

In fact, the first and the fourth assembly modes are also one surface, and a "ramp" passage is available for moving between these two different solutions of the direct kinematics problem.

References

[1] Ph. Wenger, and J. El Omri, "Changing posture for cuspidal robot manipulators," *Proceedings IEEE International Conference on Robotics and Automation*, vol. 4, pp. 3173-3178, 1996.

[2] E. Macho, O. Altuzarra, C. Pinto, and A. Hernandez, "Singularity free change of assembly mode in parallel manipulators. Application to the 3-RPR planar platform," *12th IFToMM World Congress*, Besancon, France, June 18-21, 2007, paper No. 801.

[3] C. Innocenti, and V. Parenti-Castelli, "Singularity-Free Evolution from One Configuration to Another in Serial and Fully-Parallel Manipulators," *J. Mech. Des.*, vol. 120, pp. 73-79, March 1998.

[4] M. Zein, Ph. Wenger, and D. Chablat, "An Algorithm for Computing the Cusp Points in the Joint Space of 3-R<u>P</u>R Parallel Manipulators," *The 1st European Conference on Mechanism Science (EuCoMeS)*, Obergurgl, Austria, 21-26 February, 2006.

[5] P.R. McAree, and R.W. Daniel, "An Explanation of Never-Special Assembly Changing Motions for 3-3 Parallel Manipulators," *Int. Journal of Robotics Research*, vol. 18, no. 6, pp. 556-574, June 1999.

[6] M. Zein, Ph. Wenger, and D. Chablat, "Non-singular assembly-mode changing motions for 3-RPR parallel manipulators," *Mech. Mach. Theory*, In Press, doi:10.1016/j.mechmachtheory.2007.03.011, 2007.

[7] J.J. Cervantes-Sánchez, J.C. Hernández-Rodríguez, and J.G. Rendón-Sánchez, "On the workspace, assembly configurations and singularity curves of the RRRRR-type planar manipulator," *Mech. Mach. Theory*, vol. 35, no. 8, pp. 1117-1139, 2000.

[8] N. Shvalb, H. Bamberger, and M. Shoham, "The Assembly Modes of a Planar 3RRR Robot," not published.

[9] C. Gosselin, and J. Angeles, "Singularity Analysis of Closed-Loop Kinematic Chains," *IEEE Transactions of Robotics and Automation*, vol. 6, no. 3, pp. 281-290, June 1990.

[10] L.W. Tsai, *Robot Analysis: The Mechanics of Serial and Parallel Manipulators*, New York, NY, USA, John Wiley & Sons, 1999, pp. 225-231.

[11] K.H. Hunt, and E.J.F. Primrose, "Assembly configurations of Some In-Parallel-Actuated Manipulators," *Mech. Mach. Theory*, vol. 28, no.1, pp. 31-42, 1993.

[12] M. Baili, Ph. Wenger, and D. Chablat, "A Classification of 3R Orthogonal Manipulators by the Topology of their Workspace," *IEEE International Conference on Robotics and Automation (ICRA'04)*, New Orleans, LA, USA, April 26-May 1, 2004, pp. 1933-1938.